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POSTAL BOOK PACKAGE 2025

ELECTRONICS ENGINEERING

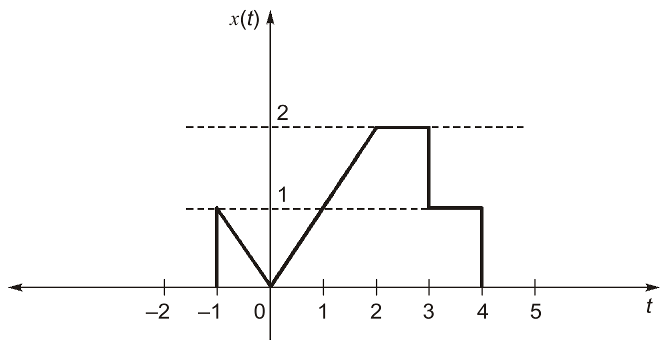
CONVENTIONAL Practice Sets

CONTENTS

SIGNALS AND SYSTEMS

1 CHAPTER Continuous Time Signal & System

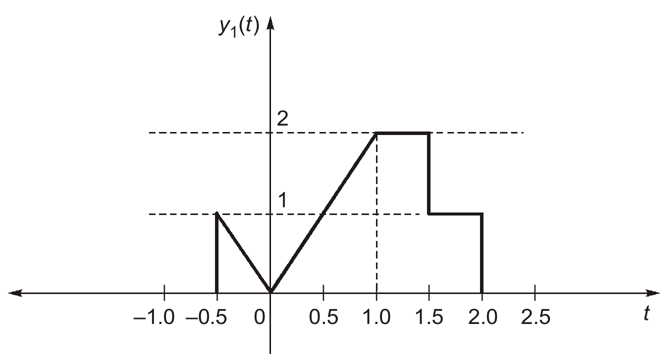
Q.1 For the given signal $x(t)$ as shown below, sketch the following signals.



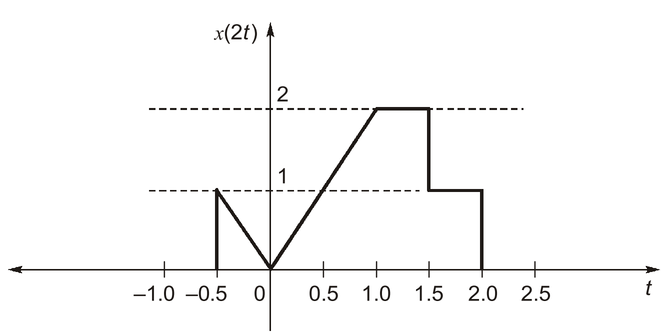
(a) $y_1(t) = x(2t)$ (b) $y_2(t) = x(2t + 4)$ (c) $y_3(t) = x\left(\frac{t}{2} + 2\right)$

Solution:

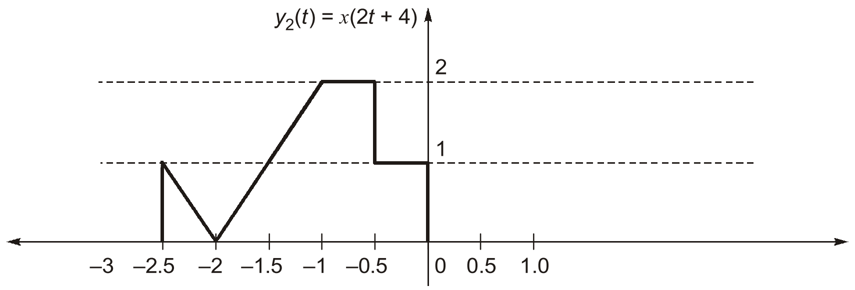
(a) We have to sketch, $y_1(t) = x(2t)$ since, $y_1(t)$ is a 2 times slowed or compressed version of $x(t)$ in time domain. So, the curve is;



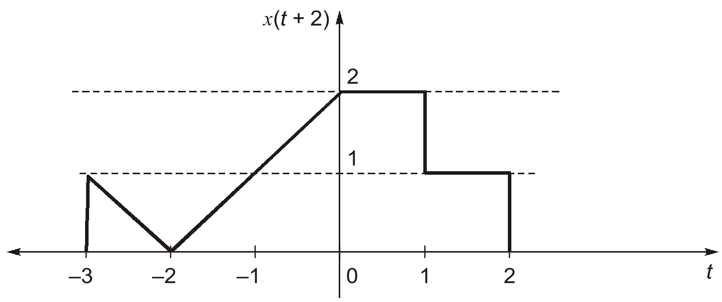
(b) We have to sketch, $y_2(t) = x(2t + 4)$ i.e. $y_2(t) = x[2(t + 2)]$.
So, we can say $y_2(t)$ is the 2 times compressed version in time of a signal which is an advance shift of 2 unit of $x(t)$.
At first we sketch the following:



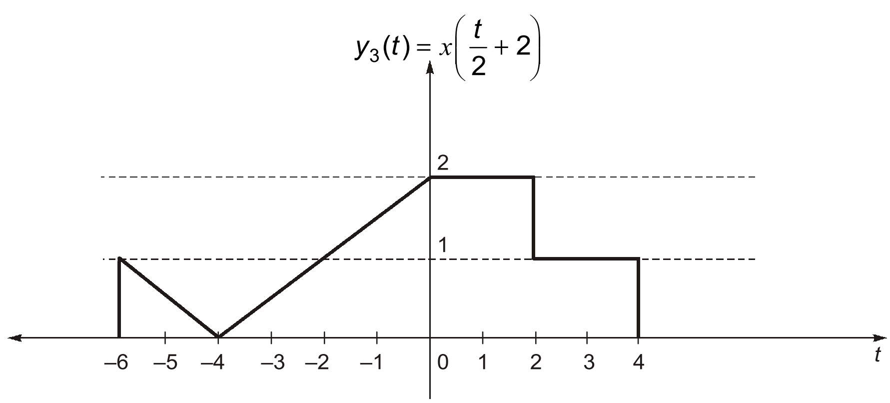
Again, we have to sketch this $x(2t)$ for an advance shift of 2 units means shift the above curve 2 unit in left side as below:



(c) We have to sketch, $y_3(t) = x(t/2 + 2)$. For this firstly, we shift 2 unit in advance shift of $x(t)$ and then expanded this signal $x(t + 2)$, by $1/(1/2) = 2$ units of the signal.



Now finally we have to sketch, $y_3(t) = x\left(\frac{t}{2}+2\right)$ as below:



Q.2 For the signals $y_1(t)$ and $y_2(t)$ shown below. Draw the differentiation of the signals and find the equations of differentiated signals.

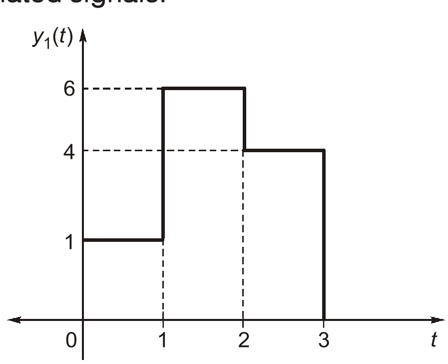


Fig. (a)

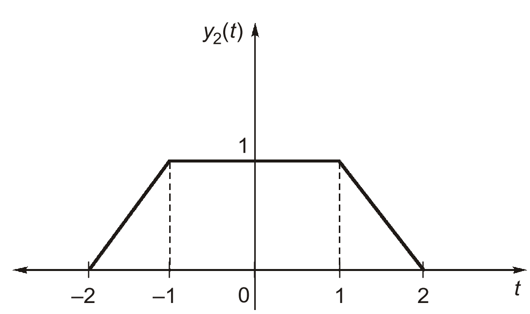


Fig. (b)

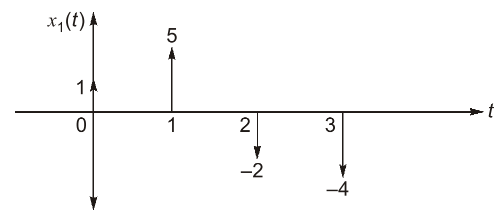
Solution:

For figure (a), we have to differentiated the signal $y_1(t)$.

Let, $$\frac{d\,y_1(t)}{dt} = x_1(t) \quad \text{...(i)}$$

Given, $$y_1(t) = u(t) + 5u(t-1) - 2u(t-2) - 4u(t-3)$$

So, from equation (i), $$x_1(t) = \frac{d\,y_1(t)}{dt} = \delta(t) + 5\delta(t-1) - 2\delta(t-2) - 4\delta(t-3)$$



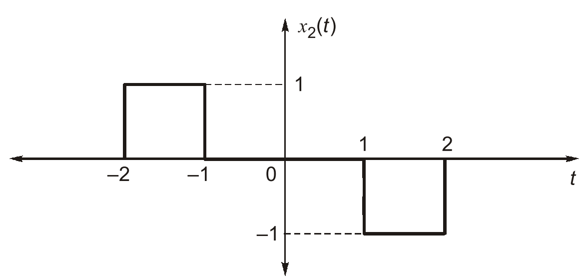
For figure (b), we have to differentiate the signal $y_2(t)$.

Let, $$\frac{d\,y_2(t)}{dt} = x_2(t) \quad \text{...(ii)}$$

Given, $$y_2(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

(where $r(t)$ represents the ramp function)

So, from equation (ii), $$x_2(t) = \frac{d\,y_2(t)}{dt} = u(t+2) - u(t+1) - u(t-1) + u(t-2)$$



Q.3 Show that the signal, $S(t) = t^{-1/4}\,u(t-1)$ is neither an energy nor a power signal.

Solution:

For an arbitrary continuous-time signal $s(t)$, the normalized energy content 'E' of $S(t)$ is defined as,

$$E = \int_{-\infty}^{\infty} |S(t)|^2\, dt \quad \text{...(i)}$$

or $$E = \int_{-\infty}^{\infty} \left|t^{-1/4}\,u(t-1)\right|^2\, dt$$

Since, $$u(t-1) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases}$$

$\therefore$ $$E = \int_{1}^{\infty} \left|t^{-1/4}\right|^2 dt = \int_{1}^{\infty} [t]^{-1/2}\, dt = \left[2[t]^{1/2}\right]_1^{\infty}$$

$\therefore$ $$E = \infty$$

Now, the normalized average power 'P' of $S(t)$ is defined as,

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |S(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|t^{-1/4}\,u(t-1)\right|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_1^T (t^{-1/4})^2\, dt = \lim_{T\to\infty} \frac{1}{2T}\int_1^T t^{-1/2}\, dt$$

$$\therefore \quad P = \lim_{T\to\infty} \frac{1}{2T}\left[2t^{1/2}\right]_1^T = \lim_{T\to\infty}\left[\frac{T^{1/2}-1}{T}\right] = \lim_{T\to\infty}\left[\frac{1-1/\sqrt{T}}{\sqrt{T}}\right] = 0$$

$$\therefore \quad P = 0$$

Here average power $P \to 0$, when total energy $E \to \infty$, which means that the condition $0 < E < \infty$ is not satisfied. Hence signal, $S(t) = t^{-1/4}\, u(t-1)$ is not an energy signal. Also, when $E \to \infty$, the value of $P \to 0$ which means that the condition $0 < P < \infty$ is not satisfied. Therefore, signal $S(t) = t^{-1/4}\, u(t-1)$ is not a power signal.

Thus, we can say that the given signal, $S(t) = t^{-1/4}\, u(t-1)$ is neither an energy signal nor a power signal. **Proved.**

Q.4 Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t) = x(t)\cos(t)$$

Check whether the is

(a) linear (b) time-invariant

Solution:

$$y(t) = x(t)\cos(t)$$

(a) To check linearity,

$$y_1(t) = x_1(t)\cos(t) \quad [y_1(t) \text{ is output for } x_1(t)]$$

$$y_2(t) = x_2(t)\cos(t) \quad [y_2(t) \text{ is output for } x_2(t)]$$

So the output for $(x_1(t) + x_2(t))$ will be

$$y(t) = [x_1(t) + x_2(t)]\cos(t)$$

$$= y_1(t) + y_2(t)$$

So the system is linear to check time invariance.

(b) The delayed output, $y(t-t_0) = x(t-t_0)\cos(t-t_0)$

The output for delayed input,

$$y(t, t_0) = x(t-t_0)\cos(t)$$

Since, $y(t-t_0) \neq y(t, t_0)$

System is time varying.

Q.5 Find out whether the system is stable/causal. If the impulse response is given by, $h(t) = e^{-6|t|}$.

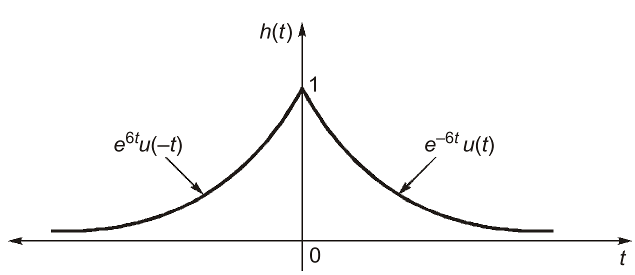
Solution:

As given that,

$$h(t) = e^{-6|t|}$$

$$\therefore \quad h(t) = e^{-6t}\cdot u(t) + e^{6t}\cdot u(-t) \quad \text{...(i)}$$

Here we see that, $h(t) \neq 0$ for $t < 0$ so, the given system is not **causal**.



For checking the system to be **"BIBO stable"**, we know that,

$$\sum_{t=-\infty}^{\infty} h(\tau)\, d\tau < \infty \quad \text{...(ii)}$$

L.H.S of equation (ii) = $\sum_{t=-\infty}^{\infty} h(\tau)\,d\tau$

$$= \sum_{t=-\infty}^{\infty}\left[e^{-6\tau}u(t)\,d\tau + e^{6\tau}\,u(-\tau)\,d\tau\right] = \int_{t=0}^{\infty} e^{-6\tau}\,d\tau + \int_{-\infty}^{0} e^{6\tau}\,d\tau$$

$$= -\frac{1}{6}\left[e^{-6\tau}\right]_0^{\infty} + \frac{1}{6}\left[e^{6\tau}\right]_{-\infty}^{0} = -\frac{1}{6}(0-1) + \frac{1}{6}(1-0)$$

L.H.S of equation (ii) = $\frac{1}{3} < \infty$

So, the system is "BIBO Stable".

Q.6 A continuous time LTI system is described by:

$$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)\,d\tau$$

Find the impulse response of the system. Is the system casual?

Solution:

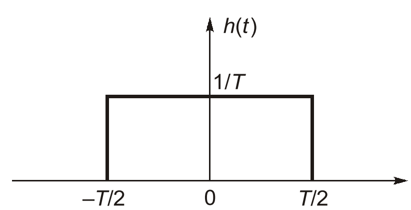
1[st] Method: $$y(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau)\,d\tau \qquad \text{...(i)}$$

Let the impulse response be $h(t)$,

$\therefore$ $$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \qquad \text{(ii)}$$

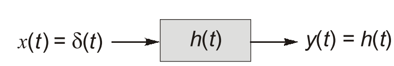
Comparing equation (i) and (ii) we get,

$$h(t) = \begin{cases} \frac{1}{T} & \frac{-T}{2} < t < \frac{T}{2} \\ 0 & \text{Otherwise} \end{cases}$$



2[nd] method:

$\therefore$ $$h(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(t)\,dt$$



$\Rightarrow$ $$h(t) = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)$$

The system is not causal as we can clearly see from equation (i) that for calculation of $y(t)$ at any time t, we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any causal system.

Q.7 Show that the following properties holds good for the derivative of $\delta(t)$.

(a) $\int_{-\infty}^{\infty} \phi(t)\,\delta'(t)\,dt = -\phi'(0)$

where, $\phi'(0) = \left.\frac{d\phi(t)}{dt}\right|_{t=0}$

(b) $t\delta'(t) = -\delta(t)$

Solution:

(a) We know that, $$\int_{-\infty}^{\infty} \phi(t)\, g^n(t)\, dt = (-1)^n \int_{-\infty}^{\infty} \phi^n(t)\, g(t)\, dt \quad \ldots\left(\phi^n(t) = \frac{d^n}{dt}\phi(t)\right) \quad \ldots(i)$$

where, $g(t)$ is the generalized function and $\phi(t)$ is the testing function.
Let, first derivative of $\delta(t) = d'(t)$ and consider, $g(t) = \delta(t)$ and $n = 1$
Putting these values in equation (i) we get,

$$\int_{-\infty}^{\infty} \phi(t)\, \delta^1(t)\, dt = (-1)^1 \int_{-\infty}^{\infty} \phi^1(t)\, \delta(t)\, dt = -\int_{-\infty}^{\infty} \frac{d\,\phi(t)}{dt} \cdot \underbrace{\delta(t)\, dt}_{\text{defined for } t = 0} = -\left.\frac{d\,\phi(t)}{dt}\right|_{t=0}$$

$$\therefore \quad \int_{-\infty}^{\infty} \phi(t)\, \delta'(t)\, dt = \phi'(0) \quad \text{Proved.}$$

(b) Let $g(t) = t\delta'(t)$ then, from equation (i) we have,

$$\int_{-\infty}^{\infty} \phi(t)[t\,\delta'(t)]\, dt = \int_{-\infty}^{\infty} [t\,\phi(t)]\,\delta'(t)\, dt$$

$$= -\frac{d}{dt}[t\,\phi(t)]\Big|_{t=0} = -[t\,\phi'(t) + \phi(t)]\Big|_{t=0}$$

$$\therefore \quad \int_{-\infty}^{\infty} \phi(t)[t\,\delta'(t)]\, dt = [\phi(0)] \quad \ldots(ii)$$

Again consider, $$\int_{-\infty}^{\infty} t\,\phi(t)\,\delta'(t)]\, dt = \int_{-\infty}^{\infty} \phi'(t)\,\delta(t)\, dt$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} \phi(t)\,[t\,\delta'(t)]\, dt = \int_{-\infty}^{\infty} \phi(t)\,[-\delta(t)]\, dt \quad \ldots(iii)$$

Thus, from the equivalence property we conclude that,

$$t\delta'(t) = -\delta(t) \quad \text{Proved.}$$

Q.8 A system has impulse response e^{-at}. What would be the response of the system, if it is excited by a delayed unit step function (delay = T)?

Solution:

$$h(t) = e^{-at}$$
$$x(t) = u(t-T)$$
$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} u(\tau - T).e^{-a(t-\tau)} d\tau$$
$$= \int_{T}^{\infty} e^{-at+a\tau} \cdot d\tau = e^{-at}\int_{T}^{\infty} e^{a\tau} d\tau = \frac{e^{-at}}{a}\left[e^{a(\infty)} - e^{aT}\right]$$

Case-I:
$a < 0$

$$\therefore \quad y(t) = -\frac{e^{-at}.e^{aT}}{a} = -\frac{e^{a(T-t)}}{a}$$

Case-II:
$a > 0$
$y(t)$ becomes unbounded in this case.